

MATH 307: Introduction to Proof

Syllabus 2021-2022

Course coordinator: Dan Dugger

Course objectives: There are two main—and quite different—objectives in this course. The first is to introduce students to the language and structures surrounding modern mathematical proof. Symbolic logic, quantifiers, set theory, functions, and induction are some of the topics in this area. Perhaps it is fair to call this the *mechanical* part of mathematical proof. The second objective is to help students become better at the problem-solving aspect of finding and creating proofs; perhaps we can call this the *non-mechanical* part. If you assign students the problem “Prove that if n is not a multiple of 3 then $n^2 - 1$ is divisible by 3”, they often are completely stuck and have no idea how to even begin—even though they understand all the words. The course should give students experience with trying examples, looking for patterns, using definitions and theorems, and so forth.

There are many approaches one could use for accomplishing these objectives, and to a large extent the exact approach is left open to the instructor. Because the course was originally designed to lead into MATH 316, I like to spend some time during the last part of the course dealing with very basic $\epsilon - \delta$ situations. This can be a matter of just understanding the logical structure in the definitions of convergence or continuity, and using it to give proofs or counterexamples in elementary situations. It is not required that this be part of the course, but I highly recommend it.

Prerequisite: MATH 247 or MATH 252 or MATH 262.

Text: There are several options here, none of them completely ideal.

- (1) D’Angelo and West, *Mathematical Thinking: Problem-Solving and Proofs*.

For many years this has been the default text. It has a huge selection of good exercises that demand ingenuity and creativity, but the downside is that it is not very easy for students to read. It does not do a great job of teaching students the mechanics of proofs, or methods for finding their own proofs. The book is also very expensive, at \$150. I personally refuse to require students to buy the book, because of the cost. I will often make use of the book myself, though, as a source for homework problems.

- (2) Sundstrom, *Mathematical reasoning and writing*.

This book is almost the opposite of D’Angelo and West. First, it is free: an electronic edition is at

<http://scholarworks.gvsu.edu/books/9/>

Second, students can read it on their own. It concentrates a lot on the mechanics of proof-writing, but to me it seems short on challenging and interesting exercises.

- (3) Gerstein, *Introduction to mathematical structures and proofs*.

(4) Taylor and Garnier, *Understanding mathematical proof*.

I personally do not like D'Angelo and West for this course at all. I have tried using Sundstrom and found it "just okay", and I ended up writing my own homework sets each week. You can find all my old homework assignments here:

http://pages.uoregon.edu/ddugger/ma307_sav.html

Exams: Two midterms and a final are typical. One midterm might be possible, but keep in mind that the students in this course need more feedback rather than less.

Workload: Weekly homework, reading, and class attendance. An average well-prepared student should expect to spend about 12 hours per week on this class (including time spent in class), but there will be a lot of variation depending on background and ability.

Active Learning: In some ways this course is a perfect candidate for an active learning classroom (one centered on in-class worksheets and student groupwork, as opposed to lecture). If you choose to go this route, David Steinberg has taught the class this way a few times and can provide information about his experience.

Warning: Keep in mind that students need a lot of feedback in this course. If the grader is the only one looking at their proofs, this is usually not sufficient. It is a good idea for the instructor to be looking over at least some of the students' work each week.

Course Learning Outcomes: Students completing this course should be able to

(1) Interpret and use logical structure in the context of proof. Examples of this are:

- If one wants to prove " $P \Rightarrow Q$ " then one proof technique is to assume P and from there deduce Q .
- If one wants to prove " P or Q " then one proof technique is to assume $\sim P$ and deduce Q .

Students should be able to define and use the biconditional, and to use de Morgan's laws. They should be able to identify the contrapositive, converse, and negation of a conditional statement, give examples, and use these concepts in proofs.

- (2) Interpret and use quantifiers. While \forall and \exists are the most important of these, it might be worth covering $\exists!$ and contexts where it shows up (for example in the context of bijective functions).
- (3) Write basic proofs, including proofs by contradiction.
- (4) Find and use counterexamples to demonstrate that statements are false.
- (5) Write induction proofs, including strong induction.

Topics and organization of the course: There are many topics that one could use as paths towards the above learning outcomes. The following is what I like to do, but instructors have a good deal of freedom in finding their own approach.

The topics I cover are

- (1) Symbolic logic
- (2) Basic set theory, including functions and their properties
- (3) Modular arithmetic
- (4) Counting and other problems in discrete mathematics
- (5) Induction
- (6) Convergence of sequences and continuity of functions (the very basics).

The philosophy behind the above choice of topics is as follows. Logic and set theory are contexts where one can study very simple proofs, but where the steps for constructing a proof are almost mechanical. I want these mechanics firmly embedded in students' minds, to serve as a foundation when they learn more advanced subjects. At the same time, this mechanical stuff is too boring—too mechanical—to spend every day on. Modular arithmetic is new for students, and fairly easy, and it gives them a context in which they can explore. For example, a homework problem might be: “Explore the values of p for which $p - 1$ is a perfect square in \mathbb{Z}_p . See if you notice patterns, and try to make a conjecture.” Subsequently we might (or might not) spend some time trying to find a proof of the conjecture. Another good homework problem is: “Explore what happens when you consider the sum of three consecutive squares modulo 3; make a conjecture and try to prove it.” There are various ways to tackle the proof, and this can lead to good discussions.

In this course I often alternate days: one day we spend on mechanical stuff, the next we spend on more exploratory stuff. After a while the two start to interact. For example, after learning the (mechanical) definition of functions being one-to-one and/or onto, the students can tackle exercises where they explore these concepts for certain functions $\mathbb{Z}_n \rightarrow \mathbb{Z}_k$.

For proofs, at the beginning I have students writing logic proofs where each step is on a separate line and carefully justified; theoretically a computer could check these for accuracy. Once we get to set theory, I have students give what I call “line proofs”. They are still written with each step on a separate line, but the structure is not quite as rigid: sometimes steps and explanations can be combined. In the final third of the course, I relax the format further and have the students writing proofs in the traditional format.

Again, the above approach will not work for everyone. Approaches to this course will vary quite a bit with personal taste and interests.

Warning: Despite the fact that there are many different approaches to this course, I want to stress that instructors should not lose sight of the main goal. Students successfully completing this course should be ready for an introductory course on analysis. In particular, students completing MATH 307 should have enough experience with quantifiers, logical reasoning, and induction to be successful in an analysis course.

Learning Environment: The University of Oregon strives for inclusive learning environments. Please notify me if the instruction or design of this course results in disability-related barriers to your participation. You are also encouraged to contact the Accessible Education Center in 360 Oregon Hall at 541-346-1155 or uoaec@uoregon.edu.

Academic Conduct: The code of student conduct and community standards is at conduct.uoregon.edu. In this course, it is appropriate to help each other on homework as long as the work you are submitting is your own and you understand it. It is not appropriate to help each other on exams, to look at other students exams, or to bring unauthorized material to exams.