Math 242: Business Calculus II

Instructor Syllabus

Fall 2021 – Summer 2022


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Course Objectives: The course-level objectives for a student completing the class are the following.

(CO 1): Use select techniques of integration to find families of antiderivatives.


(CO 3): Interpret the definite integral as an accumulation function, arithmetically and geometrically.

(CO 4): Apply methods of calculus to contexts in business, economics, and probability.

(CO 5): Expand facility with functions to functions of two or three variables.

Learning Objectives: A successful student should be able to succeed at an exam which focuses on the essential objectives (indicated by an asterisk), contains a lesser focus on supporting objectives, and may or may not contain content on the additional objectives. Remember that a single task can incorporate multiple learning objectives. (Linked Course Objectives appear in parentheses following each learning objective.)

1. *Compute an antiderivative of a polynomial, exponential, or logarithmic expression. (CO 1)

2. Apply sum and scalar multiple properties of integrals to find a family of antiderivatives. (CO 1)

3. Verify (or disprove) that a given function is a solution to a differential equation. (CO 2)

4. *Find a general solution to a simple differential equation. (CO 1, 2)

5. *Find a particular solution to a simple differential equations. (CO 1, 2)

6. Use separation of variables to find general or particular solution to a differential equation. (CO 1, 2)

7. *Use the method of substitution to evaluate an indefinite integral. (CO 1)

8. Identify Riemann sums as an approximation of definite integrals. (CO 3)

9. *Interpret the definite integral as an area of the region contained between the graph of a curve and the horizontal axis. (CO 3)

10. *Find the exact value of a definite integral using the Fundamental Theorem of Calculus. (CO 1)

11. Compute the area between two curves using integration. (CO 1, 3)

12. *Use definite integration in an applied context, including finding producer and consumer surplus, distribution of wealth, continuous income streams, and average value. (CO 1, 4)

13. *Evaluate and interpret an improper integral as a long-term sum, including within applied contexts. (CO 1, 3)

14. *Interpret input and output in a function of more than one variable within an applied context. (CO 5)

15. Find the domain of a function of two variables which is defined by formula. (CO 5)

16. Describe the level curves of a function of two variables. (CO 5)
17. *Compute partial derivatives (including using product, quotient, and chain rules) of a function of more than one variable. (CO 5)

18. Interpret the partial derivatives of a function of two variables as rates of change. (CO 5)

19. Use partial derivatives to identify substitute and complementary goods. (CO 5)

20. *Find the relative extrema of a function of two variables. (CO 5)

21. *Find the absolute extrema of a function of two variables. (CO 5)

22. *Interpret the extrema of a function defined in a non-mathematical context. (CO 4)

Most importantly, the student can model the mathematical topics described among the learning outcomes in words, then solve or simplify the relevant equations and/or expressions, and finally write a summary statement of the solution.

Additional Objectives – All additional objectives together should represent 0 – 5% of the points on an each assessment.

1. Use the method of integration by parts to evaluate an indefinite integral. (CO 1)

2. Use integration to determine whether or not a function is a continuous probability density function. (CO 1, 4)

3. Compute probabilities and expected value associated with a continuous random variable. (CO 4)

4. Employ the method of Lagrange Multipliers in finding a constrained extremum. (CO 5)

Technology: If you require a graphing calculator, use it and recommend a TI-84. If you do not allow the use of a calculator, be prepared to a) not use one yourself (lest ye be accused of hypocrisy) and b) write exams so that the simplification of arithmetically complex problems does not overshadow the actual concept they are being tested on.

Calculators like the TI-89, TI-92 and some Casio calculators (e.g. Casio FX-115ES and FX-991ES) can do differentiation and integration. The Casio calculators are not “graphing calculators”, so simply banning any calculators that graph is insufficient. You may need to be very specific about your calculator policy if you want to limit this kind of assistance on exams.
<table>
<thead>
<tr>
<th>Week</th>
<th>Sections to Cover</th>
<th>Notes</th>
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| 1    | 5.1               | 5.1 (3 hrs)  
The course starts off with virtually no (built-in) review, jumping into anti-differentiation and elementary differential equations immediately, so expect some pushback from students who are either underprepared or took 241 a while ago. |
| 2    | 5.2               | 5.2 (3 hrs)  
Relating substitution to its derivative analog (chain rule) can be helpful, but this still requires quite a bit of processing on their part, as well as explicit reminders about the differentiation involved in the substitution. |
| 3    | 5.3, 5.4          | 5.3 (2 hrs)  
There is relatively little focus on the Riemann sum in the text, so keep that in mind for your lesson planning; although 5.4 is officially the section with “applications” of the definite integral, include some word problems in 5.3 as well.  
5.4 (2 hrs)  
Units can help clear up the distinction between integrating a function. E.g. \( \int_{a}^{b} (\text{units per time}) \, d(\text{time}) = \text{units} \), whereas the average value \( V = \frac{1}{\text{units}} \cdot \int_{a}^{b} (\text{units per time}) \, d(\text{time}) = \text{units per time} \), the same as the integrand. |
| 4    | 5.4 cont’d, Exam 1| |
| 5    | 5.5               | 5.5 (3 hr)  
Consumer and producer surplus are a classic tie-in with economics courses, as are continuous income streams. |
| 6    | 6.3, 6.4          | 6.3 (1.5 hr)  
The most useful improper integrals are those of the form \( \int_{0}^{\infty} f(t) \, dt \), i.e. the “long run” trend in some function.  
6.4 (1.5 hr)  
|
| 7    | 7.1, 7.2          | 7.1 (1.5 hrs)  
There are lots of good functions of more than one variable in application, try to include a few.  
7.2 (3 hrs)  
The most confusing part of partial derivatives can just be the \( \partial \) notation; many students initially have a very hard time with the “hold this variable constant while the other changes” process. |
| 8    | 7.2 cont’d; Exam  | Try to have a midterm exam this week so that students have feedback before the week 7 drop deadline. |
| 9    | 7.3               | 7.3 (3 hrs)  
Absolute extrema on a closed, bounded region can be very time-consuming, so plan accordingly if you discuss the topic. |
| 10   | 7.3 cont’d; 7.5*   | 7.5* (0 – 1.5 hrs)  
Catch-up and review  
It can be difficult to find a balance between exercises that you want to use Lagrange Multipliers for (as opposed to direct substitution) and those that are manageable using Lagrange Multipliers.  
If giving a non-cumulative exam worth at most 20% of the course grade, you may do so during week 10. |
Additional Notes:

- **Alternative Schedule:** I prefer to begin the course with Chapter 7, as it will review differentiation strategies and is in some ways an easier way to ramp up into anti-differentiation. Chapter 7 takes up the first three weeks (I don’t cover sections 7.4, 7.5), then return to chapters 5 and 6 to close out the course. This would permit a chapter 7 test earlier in the quarter (e.g. week 4) and then a midterm and final over chapters 5 and 6.

- The typical consumer of this course is either an economics student who doesn’t want to take Math 252, or a pre-business major who plans to have a more intensive mathematics curriculum in their business degree. In spring 2019, the business school removed the requirement that *all* pre-business students take Math 242 as part of their admission requirements.

- Common areas of difficulty: Basic algebra (factoring, simplifying and operations on fractions), chain rule, logarithms, applications of any sort, modeling mathematically in particular. Be conscious of these facts when you approach each topic so that you can be ready for the confused looks, frustrated sighs, and eye rolling. Combat them with detailed examples and ample opportunities for practice. Basic algebra review is most effective when integrated into new concepts, so do it on an as-needed basis. Students complain about the abstract problems because they arent relatable. Students complain about word problems because they’re hard. It’s a difficult situation to win, but a responsible math class for predominantly non-majors involves both abstract mathematics and applications.

- It is acceptable to give three midterm exams rather than two midterm exams and a cumulative final. The content from Chapters 5 and 6 is relatively distinct from Chapter 7, and unlike 241, 242 is a terminal course and not used as a prerequisite for any further classes. I prefer a midterm for Chapter 7, a midterm for sections 5.1 – 5.3, and then a final exam over Chapters 5 and 6.

- Some instructors choose to cover 6.1 (integration by parts). That’s a reasonable choice, although the business school has identified it as not a priority. My halfway measure is to define \( \int te^{kt} \, dt \) explicitly and then skip integration by parts, as this is typically the only integral of use in business applications anyway.

- Word problems should be a key feature of the course. Consider introducing new topics in a non-mathematical context (there is lots of evidence that this helps students learn the material to begin with, but also to retain it longer). Differential equations are an especially powerful stage on which to present business and economics phenomena in context.

- Mike has lecture guides, worksheets, quizzes, exams, practice packets, and links to screen capture lecture videos available (they are located at the YouTube channel “mikesmathchannel”).

- Research in instruction is clear: small, regular assessments that count (a very small amount) toward the course grade encourage students to more accurately determine their standing in the course prior to a large assessment like a midterm. Even if you plan to primarily lecture in class, please consider including 1 – 5 minutes of regular student work time to answer quick computational
or conceptual questions during class. Mike’s clicker questions are an example of this (20 minutes total per week, participation points for engaging, bonus points for accuracy, grading done by iClicker system).