Oral Exam Syllabus - Rob Muth

Books

1. J. A. Green, *Polynomial Representations of* GL_n, Second edition, Springer-Verlag 2007.

   Chapters 1-6 (full text of the first edition). Polynomial representations of GL_n; the Schur algebra; contravariant duality; weights and characters of representations; the modules $D_{\lambda,K}$ and $V_{\lambda,K}$ and basis theorems; representation theory of S_n via the Schur functor; Specht modules and their duals.

2. J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory*, Springer-Verlag 1972.

   Chapters I-III, V, VI. Theorems of Engel, Lie, and Weyl; the Killing form; classification of semisimple Lie algebras; root systems; universal enveloping algebras and the PBW theorem; Verma modules; finite dimensional representations of semisimple Lie algebras; Harish-Chandra Theorem; Weyl character formula and the Kostant multiplicity formula.

3. G. D. James, *The Representation Theory of the Symmetric Groups*, Springer-Verlag 1978.

   Full text. Specht modules and basis theorem; branching theorem; irreducible representations of S_n; the rules of Young, Littlewood-Richardson, Murnaghan-Nakayama; hook formula; Young's orthogonal form.

4. A. Kleshchev, *Linear and Projective Representations of Symmetric Groups*, Cambridge University Press 2005.

   Chapters 1-5. Okounkov-Vershik approach to representation theory of S_n in characteristic 0; Gelfand-Zetlin bases and formulas of Young and Murnaghan-Nakayama; the degenerate affine Hecke algebra $\mathcal{H}_n$; Mackey theorem; characters of $\mathcal{H}_n$-modules; crystal operators $\tilde{e}_a$, $\tilde{f}_a$ and the function ε_a.

Papers

1. M. Khovanov, A. Lauda, *A diagrammatic approach to categorification of quantum groups I*, `arXiv:0803.4121 [math.QA]` 5 May 2008.

   Definition of the KLR-algebra R_α; diagrammatic calculus; nilHecke rings and their representations; center and basis theorem for R_α; representations of R_α and their characters; induction and restriction functors; Mackey theorem; isomorphism of $[\mathrm{Proj}(R_\alpha)]$ and Lusztig's $\mathbf{f}_\mathcal{A}$; injectivity of character map.

2. A. Kleshchev, A. Ram, *Homogeneous representations of Khovanov-Lauda algebras*, `arXiv:0809.0557 [math.RT]` 23 Sep 2008.

   Calibrated representations of R_α; classification and construction of homogeneous modules for R_α of simply-laced type; minuscule elements of the Weyl group, skew shapes, and connection to the hook formula.

3. A. Kleshchev, A. Ram, *Representations of Khovanov-Lauda-Rouquier algebras and combinatorics of Lyndon words*, `arXiv:0909.1984 [math.RT]` 10 Sep 2009.

   Representation theory of the KLR-algebra R_α; pairing and duality in $\mathrm{Proj}(R_\alpha)$ and $\mathrm{Rep}(R_\alpha)$; categorification of Lusztig's $\mathbf{f}_\mathcal{A}$ and $\mathbf{f}^*_\mathcal{A}$; cuspidal modules and classification of irreducible modules for finite type.

4. A. Kleshchev, *Cuspidal systems for affine Khovanov-Lauda-Rouquier algebras*, `arXiv:1210.6556 [math.RT]` 24 Oct 2012.

   Convex orders on positive roots; cuspidal systems; classification of irreducible R_α-modules for affine type; extremal words; standard modules; imaginary modules; minuscule imaginary modules in simply-laced type.

Classes

1. Math 681-682-683, Fall 2012-Spring 2013, Representation Theory, V. Ostrik, A. Kleshchev.

   Representation theory of Lie algebras as described in Humphreys section; quivers and path algebras and their representations; Gabriel's theorem; theory of Euclidean diagrams, including translate functors, preprojective, preinjective and regular modules; Hall algebras.

2. Math 681-682, Fall 2013-Winter 2014, Commutative Algebra and Algebraic Geometry, A. Polishchuk.

3. Math 607, Fall 2013, Quantum Groups, J. Brundan.

4. Math 607, Winter 2014, Infinite Dimensional Lie Algebras, V. Ostrik.