
Course Goals: A student successfully completing the course should, in general, have a foundation in non-trigonometric integral calculus, elementary differential equations, and introductory multivariable calculus. The student can model the mathematical topics described among the learning outcomes in words, then solve or simplify the relevant equations and/or expressions, and finally write a summary statement of the solution. In short, all of the learning outcomes should be incorporated with skill at mathematical modeling.

Learning Outcomes: These outcomes should all be represented on exams (not necessarily a single, cumulative final examination) in your course. A successful student can...

- find antiderivatives of polynomial, exponential, and logarithmic expressions.
- use substitution to evaluate indefinite integrals.
- find particular solutions to simple differential equations.
- use separation of variables to find general or particular solutions to differential equations.
- identify Riemann sums as an approximation of definite integrals.
- relate definite integrals to the area between a curve and the horizontal axis.
- find the exact value of definite integrals using the Fundamental Theorem of Calculus.
- determine area between curves using integration.
- use definite integration to evaluate applications to business and economics, including producer and consumer surplus, distribution of wealth, continuous income streams, and average value.
- evaluate and interpret improper integrals in context.
- use integration to determine whether or not functions are continuous probability density functions.
- compute probabilities and expected value associated with continuous random variables.
- interpret input and output in functions of more than one variable.
- evaluate and find the domain of functions of two variables.
- identify and/or graph level curves of functions of two variables.
- compute partial derivatives (including using product, quotient, and chain rules) of functions of more than one variable.
- interpret as rates of change the partial derivatives of functions of two variables.
- use partial derivatives to identify substitute and complementary goods.
- find relative extrema of functions of two variables.
- find and interpret absolute extrema of functions defined in non-mathematical contexts.
- (optional) employ the method of Lagrange Multipliers in finding a constrained extremum.

Most importantly, the student can model the mathematical topics described among the learning outcomes in words, then solve or simplify the relevant equations and/or expressions, and finally write a summary statement of the solution.
**Technology:** If you require a graphing calculator, use it and recommend a TI-84. If you do not allow the use of a calculator, be prepared to a) not use one yourself (lest ye be accused of hypocrisy) and b) write exams so that the simplification of arithmetically complex problems does not overshadow the actual concept they are being tested on.

Calculators like the TI-89, TI-92 and some Casio calculators (e.g. Casio FX-115ES and FX-991ES) can do differentiation and integration. The Casio calculators are not “graphing calculators”, so simply banning any calculators that graph is insufficient. You may need to be very specific about your calculator policy if you want to limit this kind of assistance on exams.

<table>
<thead>
<tr>
<th>Week</th>
<th>Sections to Cover</th>
<th>Notes</th>
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<tbody>
<tr>
<td>1</td>
<td>5.1</td>
<td>5.1 (3 hrs) The course starts off with virtually no (built-in) review, jumping into anti-differentiation and elementary differential equations immediately, so expect some pushback from students who are either underprepared or took 241 a while ago.</td>
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<td>2</td>
<td>5.2</td>
<td>5.2 (3 hrs) Relating substitution to its derivative analog (chain rule) can be helpful, but this still requires quite a bit of processing on their part, as well as explicit reminders about the differentiation involved in the substitution.</td>
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<td>3</td>
<td>5.3, 5.4</td>
<td>5.3 (2 hrs) There is relatively little focus on the Riemann sum in the text, so keep that in mind for your lesson planning; although 5.4 is officially the section with “applications” of the definite integral, include some word problems in 5.3 as well. 5.4 (2 hrs) Units can help clear up the distinction between integrating a function. E.g. ( \int_a^b (\text{units per time}) , d(\text{time}) = \text{units} ), whereas the average value ( V = \frac{1}{\text{units}} \int_a^b (\text{units per time}) , d(\text{time}) = \text{units per time} ), the same as the integrand.</td>
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<td>4</td>
<td>5.4 cont’d, Exam 1</td>
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<td>5</td>
<td>5.5</td>
<td>5.5 (3 hr) Consumer and producer surplus are a classic tie-in with economics courses, as are continuous income streams.</td>
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<td>6</td>
<td>6.3, 6.4</td>
<td>6.3 (1.5 hr) The most useful improper integrals are those of the form ( \int_0^\infty f(t) , dt ), i.e. the “long run” trend in some function. 6.4 (1.5 hr)</td>
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<tr>
<td>7</td>
<td>7.1, 7.2</td>
<td>7.1 (1.5 hrs) There are lots of good functions of more than one variable in application, try to include a few. 7.2 (3 hrs) The most confusing part of partial derivatives can just be the ( \partial ) notation; many students initially have a very hard time with the “hold this variable constant while the other changes” process.</td>
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<tr>
<td>8</td>
<td>7.2 cont’d; Exam</td>
<td>Try to have a midterm exam this week so that students have feedback before the week 7 drop deadline.</td>
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7.3 (3 hrs) Absolute extrema on a closed, bounded region can be very time-consuming, so plan accordingly if you discuss the topic.

7.3 cont’d; 7.5*, 7.6*  
7.5* (0 – 1.5 hrs) It can be difficult to find a balance between exercises that you want to use Lagrange Multipliers for (as opposed to direct substitution) and those that are manageable using Lagrange Multipliers.

7.6* (0 – 1.5 hrs) Double integrals are not critical material, but if you have the time they are good additional practice in integration. If giving a non-cumulative exam worth at most 20% of the course grade, you may do so during week 10.

Catch-up and review  

### Additional Notes:

- **Alternative Schedule:** There is an argument to beginning the course with Chapter 7, as it will review differentiation strategies and is in some ways an easier way to ramp up into antiderivatives. If you plan to cover 7.6 (double integrals), this may not be wise unless you return to that section after completing Chapter 6. Begin the course with Chapter 7 in the first three weeks, then return to chapters 5 and 6 to close out the course. This would permit a chapter 7 test earlier in the quarter (e.g. week 4) and then a midterm and final over chapters 5 and 6. I tried this reorganization in winter 2018 and I feel it worked out quite well.

- The typical consumer of this course is a pre-business major satisfying their mathematics requirement. They will need 241, 242, and 243 completed for a grade in order to apply to the business school. More than any other math class, these students can be resentful of the need to take the course. There are also a sizable number of economics students who take this class instead of 252.

- **Common areas of difficulty:** Basic algebra (factoring, simplifying and operations on fractions), chain rule, logarithms, applications of any sort, modeling mathematically in particular. Be conscious of these facts when you approach each topic so that you can be ready for the confused looks, frustrated sighs, and eye rolling. Combat them with detailed examples and ample opportunities for practice. Basic algebra review is most effective when integrated into new concepts, so do it on an as-needed basis. Students complain about the abstract problems because they aren’t relatable. Students complain about word problems because they’re hard. It’s a difficult situation to win, but a responsible math class for predominantly non-majors involves both abstract mathematics and applications.

- It is acceptable to give three midterm exams rather than two midterm exams and a cumulative final. The content from Chapters 5 and 6 is relatively distinct from Chapter 7, and unlike 241, 242 is a terminal course and not used as a prerequisite for any further classes.

- Some instructors choose to cover 6.1 (integration by parts). That’s a reasonable choice, although the business school has identified it as not a priority. My halfway measure is to define \[ \int te^{kt} \, dt \] explicitly and then skip integration by parts, as this is typically the only integral of use in business applications anyway.
• Word problems should be a key feature of the course. Consider introducing new topics in a non-mathematical context (there is lots of evidence that this helps students learn the material to begin with, but also to retain it longer). Differential equations are an especially powerful stage on which to present business and economics phenomena in context.

• Mike has lecture guides, worksheets, quizzes, exams, practice packets, and lecture videos (on YouTube at [https://www.youtube.com/channel/UCbjeLRIWP_dJB3i14QDDitg](https://www.youtube.com/channel/UCbjeLRIWP_dJB3i14QDDitg)) available upon request.

• Research in instruction is clear: small, regular assessments that count (a very small amount) toward the course grade encourage students to more accurately determine their standing in the course prior to a large assessment like a midterm. Even if you plan to primarily lecture in class, please consider including 1–5 minutes of regular student work time to answer quick computational or conceptual questions during class. Mike’s clicker questions are an example of this (20 minutes total per week, participation points for engaging, bonus points for accuracy, grading done by iClicker system).