ALGEBRA QUALIFYING EXAMINATION, FALL 2014

Your Name:

Part I. Carefully state each of the following (6 points each):

1. Explain what are unit and counit of adjunction and what do they have to do with adjoint functors.

2. Wedderburn-Artin Theorem.

3. Orthogonality relations (row and column) for irreducible characters of finite groups.

Part II. True or false? If true provide a brief explanation, if false provide a counterexample (8 points each):

1. Let I, J be ideals in $F[T_1, \ldots, T_n]$. Then $\mathcal{V}(IJ) = \mathcal{V}(I \cap J)$.

2. Let R be a commutative ring and $I, J \lhd R$ be two ideals of R. If the modules R/I and R/J are isomorphic then $I = J$.

3. If R is a ring with no non-trivial left ideals, then it also has no non-trivial right ideals.

4. $\mathbb{Z}_3 \otimes_{\mathbb{Z}} (\mathbb{Z} \otimes_{\mathbb{Z}} \mathbb{Z}_2) \cong \mathbb{Z}_6$.

5. A morphism of affine algebraic sets which is a homeomorphism is an isomorphism

Part III. Prove the following statements (14 points each):

1. If $R \subseteq A$ is an integral extension of noetherian rings then $\dim R = \dim A$.

2. Let A be an $n \times n$ matrix over a field F. Then A is similar to its transpose A^t.

3. Let G be a finite group with precisely five inequivalent irreducible representations of dimensions $1, 1, 2, 3$ and d. Find d.