Suppose that $\mathcal{F}$ is a separable Banach space for all $n \in \mathbb{Z}^+$. Prove or disprove: is $\mathcal{F}$ a Banach space if $\mathcal{F}$ is a Banach space?

In the following you do not need to prove that $\mathcal{F}$ is a vector space or that the line segment $[x, y]$ is a line segment in $\mathcal{F}$.

Let $\mathcal{F}$ be a sequence of linear vector spaces. Define $\mathcal{F}$ to be the

\[ \mathcal{F} = \left\{ (x)_{n \in \mathbb{N}} : (x_n) \right\} \]

and such that $\lim_{n \to \infty} x_n = 0$ with the norm:

\[ \| \cdot \|_{\mathcal{F}} = \left( \sum_{n=0}^{\infty} \| x_n \|^2 \right)^{1/2} \]

and such that $\lim_{n \to \infty} x_n = 0$ for all $x_n \in \mathcal{F}$ for all $n \in \mathbb{N}$.

1. Let $\{ \alpha_n \}_{n=0}^{\infty} \subseteq \mathbb{N}$ be a sequence of natural numbers. Define $\mathcal{F}$ to be the

\[ \mathcal{F} = \left\{ (x)_{n \in \mathbb{N}} : (x_n) \right\} \]

where we have

\[ \lim_{n \to \infty} x_n = 0 \quad \text{for all } x_n \in \mathcal{F} \quad \text{for all } n \in \mathbb{N}. \]

2. Let $\{ \alpha_n \}_{n=0}^{\infty} \subseteq \mathbb{N}$ be a sequence of natural numbers. Assume that

\[ \lim_{n \to \infty} x_n = 0 \quad \text{for all } x_n \in \mathcal{F} \quad \text{for all } n \in \mathbb{N}. \]

3. Let $\mathcal{F}$ be a sequence of linear vector spaces. Define $\mathcal{F}$ to be the

\[ \mathcal{F} = \left\{ (x)_{n \in \mathbb{N}} : (x_n) \right\} \]

where we have

\[ \lim_{n \to \infty} x_n = 0 \quad \text{for all } x_n \in \mathcal{F} \quad \text{for all } n \in \mathbb{N}. \]

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\[ \frac{e^z}{z - (z)f} \]