University of Oregon
Department of Mathematics

2005 Niven Lectures

March 28 – April 1, 2005

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A tea will precede Lectures 1 and 3 at 3:15 p.m. in Fenton 219, The Faculty and Graduate Lounge. A reception will follow the Monday lecture in 219 Fenton.

Lecture 1: A tale on Two Fractals

4 p.m., Monday, March 28, 106 Deady Hall

Abstract: The physical notion of a fractal set and its mathematical counterparts. Two examples: Sierpinski gasket S and Apollonian gasket A. Harmonic functions on S and generalized numerical systems. Geometric, group-theoretic, and arithmetic properties of A. Are S and A similar?

Lecture 2: Self-Similar Fractal Sets and Generalized Numerical Systems (Undergraduate Lecture)

12 p.m., Wednesday, March 30, 229 McKenzie Hall

Abstract: What is and what can be a numerical system? Usually, to encode a real number we use an infinite sequence $a = (a_1, a_2, a_3, ...)$ where all "digits" a_i take values from some set A. For example, we can put $A = \{0, 1\}$ and associate to a sequence a the number $\mathrm{val}(a) = \sum_{k>0} a_k / 2^k$ (the standard binary system). If we keep $A = \{0, 1\}$ and replace 2 by some exotic base b (e.g. by –2 or by $1 + i$, the set of possible values val(a) can be a fractal.

There are quite different ways to associate a number val(a) to a sequence a. For instance, continuous fractions $a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{a_4 + \cdots}}}$ give such a way.

It is interesting that all these "numerical systems" can be considered as particular cases of a general scheme which we call matrix numerical systems. A new class of function arises when we use these generalized numerical systems. Namely, we can write a number x in one system and then read it in another one. We get a functions $x \mapsto y$, which usually can not be expressed in terms of known elementary functions. As examples, I mention the "question function" of Minkowski and harmonic functions on the Sierpinski gasket.

Lecture 3: Descartes Theorem and its Generalization (Undergraduate Lecture) 4 p.m., Friday, April 1, 106 Deady

Abstract: It is clear that three pairwise tangent discs on a plane can have arbitrary radii r_1, r_2, r_3. But for a fourth disc, tangent to these three, the radius r_4 must satisfy some equations. This equation was first discovered by René Descartes in the XVII century and impressed many people, even non-mathematicians. It turns out that this equation admits two nice reformulations: in terms of Hermitian 2×2 matrices and in terms of space-like vectors in special relativity.

These reformulations allow not only to give a "natural" proof of Descartes' equation, but also essentially generalize it. The filling of a unit disc by discs with integral boundary curvatures, arising here, leads to several beautiful geometric, group-theoretic, and arithmetic questions which are mostly unsolved.