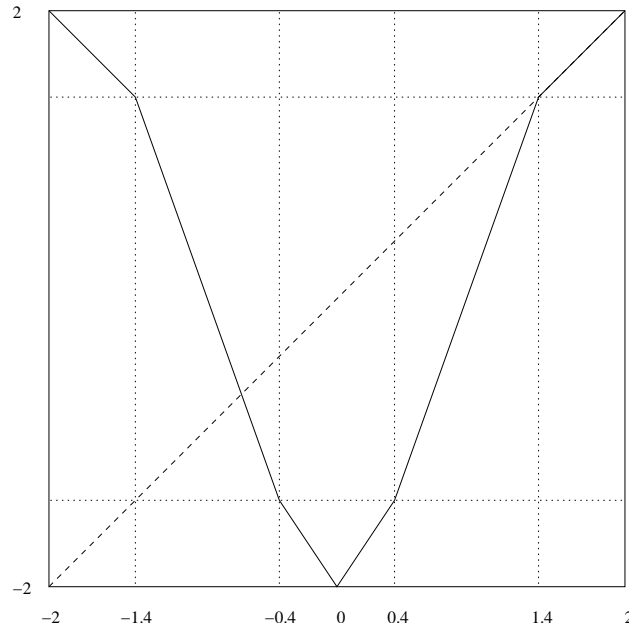

Math 457
Homework Due Wednesday, May 14

1. Consider the function $f: [-2, 2] \rightarrow [-2, 2]$ whose graph is below.



Determine what happens to the orbit of every point $x \in [-2, 2]$. Compare and contrast the dynamical behavior with that of the function $Q_{-2}(x) = x^2 - 2$, whose graph is somewhat similar.

2. Let $f(x) = 5x^2 - 2x + 3$. Determine a linear change-of-coordinates $\varphi(x) = Ax + B$ giving a conjugacy between $f(x)$ and a function of the form $g(x) = x^2 + C$. You must determine A , B , and C .
3. Let $f(x) = \lambda x(1 - x)$ where $\lambda > 0$. Determine a linear change of coordinates $\varphi(x) = Ax + B$ giving a conjugacy between $f(x)$ and a function of the form $g(x) = x^2 + C$. You must determine the values of A , B , and C (in terms of λ).

When you did Experiment 8.3 you noticed that the orbit diagrams of the logistic family and the quadratic family were very similar (only reversed, and some proportions slightly different). The above change-of-coordinates explains these similarities.

4. This problem concerns the dynamical behavior of $g(u) = u^2 - 2$.
- (a) Verify that $\left\{2 \cos\left(\frac{2}{5}\pi\right), 2 \cos\left(\frac{4}{5}\pi\right)\right\}$ is a 2-cycle for g . Use the trig identity $\cos(2\theta) = 2 \cos(\theta)^2 - 1$.
- (b) We saw in class that the function $\varphi(x) = 2 \cos(\pi x)$ gives a change-of-coordinates converting the tent function from problem #2 into $g(x) = x^2 - 2$. By finding three rational numbers which form a 3-cycle for the tent function (which is hopefully easy by now, we've done it so many times), determine an explicit 3-cycle for g .
5. Explain how you would find an $x \in [0, 1]$ whose orbit under T_2 is **dense**. (This means that for every $y \in [0, 1]$ and every $\epsilon > 0$, there is an $n > 0$ such that $d(y, T_2^n(x)) < \epsilon$; or in English, the orbit of x comes arbitrarily close to every point in $[0, 1]$.) Find the first nine digits of such an x .
6. Do the following exercises from Chapter 15: 1a, 2ce, 3df, 5acd, 8ab, 9.