
Math 457
Homework Due Wednesday, April 30

1. Let K be the Cantor set, and consider the following two claims:

- (i) If a and b are two points in the Cantor set, then there is a point between a and b which is also in the Cantor set.
- (ii) If a and b are two points which are *not* in the Cantor set, then there is a point between a and b which is also not in K .

The first statement is FALSE—explain why by giving a counterexample. The second statement is TRUE—explain why. (Hint for second part: Assume $a < b$, then look at the first digit where their ternary expansions differ).

2. Let x be the real number whose ternary expansion is $0.022201\overline{20}$. Determine an $\epsilon > 0$ with the property that no element in $(x - \epsilon, x + \epsilon)$ belongs to the Cantor set (and explain your answer). (Hint: Choose ϵ to be $1/3^n$ for a certain value of n).

3. If $d \in \{0, 1, 2\}$, let $\tilde{d} = 2 - d$. Define a function $E: [0, 1] \rightarrow [0, 1]$ by

$$E(0.0a_1a_2a_3\dots) = 0.\tilde{a}_1\tilde{a}_2\tilde{a}_3\dots \quad E(0.1a_1a_2a_3\dots) = 0.0\tilde{a}_1\tilde{a}_2\tilde{a}_3\dots \quad E(0.2a_1a_2a_3\dots) = 0.a_1a_2a_3\dots$$

where the expansions are in base 3. Determine base 3 expansions for all the fixed points and two-cycles of E .

4. Let $S = \mathbb{Z} \times \mathbb{Z} = \{(n, m) \mid n, m \in \mathbb{Z}\}$. Explain why S is a countable set. (Hint: One way involves picturing S as the set of lattice points in the plane).

5. Do Chapter 8, questions 4, 5 (don't hand in anything for question #4). The website

<http://math.bu.edu/DYSYS/applets/OrbitDgm.html>

will sketch the orbit diagram for you.

6. Do Experiment 8.3 from Chapter 8. Hand in your observations about the periods of the windows.