
Math 457
Homework Due Friday, May 27

For the following questions you should use one of the websites listed on the course webpage for exploring the Mandelbrot set. The following two seem to be the most useful:

- (A) <http://aleph0.clarku.edu/~djoyce/julia/explorer.html>
(B) <http://math.bu.edu/DYSYS/applets/JuliaIteration.html>

1. Look at the Julia sets in Figures 17.8a–h after page 258 in your book. By playing around, find the (approximate) c values that correspond to each of them.

Website (B) is probably best for this task, but note that there is a slight flaw in this website: when you click on a point in the Mandelbrot set below the real axis, the c value the website gives you has its real part incorrect. You can generally compensate for this by noticing that the Mandelbrot set is symmetric about the real axis.

2. The Mandelbrot set has various “bulbs” or “decorations” appearing in it. Some of these are attached directly to the main cardioid, some of them are located elsewhere. See the diagram on page 257 for some of them. Each of these bulbs corresponds to c -values for which Q_c has an attractive cycle.

Use website (B), and click on a c -value inside the main cardioid. The corresponding Julia set is then shown in the diagram below the Mandelbrot set. The white dot jumping around is the orbit of 0 under Q_c . You will notice that it quickly stabilizes—the orbit converges to a fixed point. In fact, the main cardioid is precisely the region of all c -values where an attractive fixed point exists.

Now click on c -values inside some of the bulbs attached to the main cardioid. You will notice that the orbit of 0 converges to a cycle, and you can figure out the period by watching it bounce around. All c -values within a given bulb have attractive cycles of the same period. By experimenting with the website, find the periods of at least 20 bulbs attached to the main cardioid. Draw a diagram of the main cardioid showing the bulbs and the periods you found. Do you notice any patterns?

3. There is a period 3 bulb attached to the very top of the main cardioid. Zoom in on the “antennas” at the very top of this bulb. You will see some dark regions which look like tiny copies of the Mandelbrot set—they will look more like tiny copies if you increase the number of iterations your website is performing. These are also examples of ‘decorations’ in the Mandelbrot set. Click on c -values in these regions and again determine the periods of some of them (you will have to focus your brain when counting the period!). Draw a diagram showing what you found. Repeat this experiment for one of the other bulbs attached to the main cardioid.

Incidentally, if you count the number of antennas attached to each bulb on the main cardioid you will notice something. What is it?

4. Try to find all the decorations of periods 3, 4, and 5. Some are attached to the main cardioid, some are not. The diagram on page 257 shows some of them. There are exactly three period 3 decorations, six period 4 decorations, and 15 period five decorations. We will talk about where these numbers come from in class. Find all the period 3 and period 4 decorations. It’s hard to find all the period 5 ones, but try to do it.
5. Pick a bulb attached to the main cardioid and zoom in on its antennas. You will notice that one of the antennas is shorter than the others. Now click on a c -value in the bulb, and look at the orbit of 0 (again, website (B) is probably best for this). There is a correspondence between the behavior of this orbit and the placement of the antenna. Can you figure out what it is?