A tea will precede Lectures 1 and 3 at 3:15 p.m. in Fenton 219, The Faculty and Graduate Lounge. A reception will follow the Monday lecture in 219 Fenton.

**Lecture 1: A tale on Two Fractals**

*4 p.m., Monday, March 28, 106 Deady Hall*


**Lecture 2: Self-Similar Fractal Sets and Generalized Numerical Systems (Undergraduate Lecture)**

*12 p.m., Wednesday, March 30, 229 McKenzie Hall*

Abstract: What is and what can be a numerical system? Usually, to encode a real number we use an infinite sequence $a = (a_1, a_2, a_3, \ldots)$ where all "digits" $a_i$ take values from some set $A$. For example, we can put $A = \{0, 1\}$ and associate to a sequence $a$ the number $\text{val}(a) = \sum_{k>0} a_k / 2^k$ (the standard binary system). If we keep $A = \{0, 1\}$ and replace 2 by some exotic base $b$ (e.g. by $-2$ or by $1 + i$), the set of possible values $\text{val}(a)$ can be a fractal.

There are quite different ways to associate a number $\text{val}(a)$ to a sequence $a$. For instance, continuous fractions

\[
a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{a_4 + \ldots}}}\]

It is interesting that all these “numerical systems” can be considered as particular cases of a general scheme which we call matrix numerical systems. A new class of function arises when we use these generalized numerical systems. Namely, we can write a number $x$ in one system and then read it in another one. We get a functions $x \mapsto y$, which usually cannot be expressed in terms of known elementary functions. As examples, I mention the “question function” of Minkowski and harmonic functions on the Sierpinski gasket.
Lecture 3: Descartes Theorem and its Generalization (Undergraduate Lecture) 4 p.m., Friday, April 1, 106 Deady

Abstract: It is clear that three pairwise tangent discs on a plane can have arbitrary radii \( r_1, r_2, r_3 \). But for a fourth disc, tangent to these three, the radius \( r_4 \) must satisfy some equations. This equation was first discovered by René Descartes in the XVII century and impressed many people, even non-mathematicians. It turns out that this equation admits two nice reformulations: in terms of Hermitian \( 2 \times 2 \) matrices and in terms of space-like vectors in special relativity.

These reformulations allow not only to give a “natural” proof of Descartes’ equation, but also essentially generalize it. The filling of a unit disc by discs with integral boundary curvatures, arising here, leads to several beautiful geometric, group-theoretic, and arithmetic questions which are mostly unsolved.