1. Suppose that \( \{f_n\}_{n \in \mathbb{N}} \) is a sequence of continuous functions on \([0, 1]\), which is converging uniformly. Determine whether the limit below exists
\[
\lim_{n \to \infty} \int_0^1 n f_n(t) 2^{-nt} \, dt.
\]
If yes, then find its value.

2. Give an example of a dense \( G_\delta \) set \( E \subset [0, 1] \) such that \(|E| = 0\).

3. Let \( \mu \) be a positive measure on \( X \). Prove that
\[
0 < r < p < s < \infty \implies L^r(\mu) \cap L^s(\mu) \subset L^p(\mu).
\]

4. Let \( f_a(x) = \exp(- (x-a)^2/2) \). Prove that the linear combinations of \( \{f_a : a \in \mathbb{R}\} \) are dense in \( L^2(\mathbb{R}) \).

5. Let \( \mu \) be a positive \( \sigma \)-finite measure and \( 1 \leq p < \infty \). Let \( f \) be a measurable function. Prove that a multiplication operator \( M_f : L^p(\mu) \to L^p(\mu) \), \( M_f(g) = fg \), is bounded \( \iff \|f\|_\infty < \infty \).

6. Suppose that \( f : [a, b] \to \mathbb{R} \) is AC (absolutely continuous). Prove that that there exist non-decreasing AC functions \( f_1, f_2 : [a, b] \to \mathbb{R} \) such that \( f = f_1 - f_2 \).

7. Find a bounded linear functional \( L \) on \( L^p(\mathbb{R}) \), \( 1 \leq p < \infty \), such that
\[
L(\chi_{[k, k+1]}) = (1 + |k|)^{-1} \quad \text{for all } k \in \mathbb{Z}.
\]

8. Let \( g \) be a meromorphic function on a simply connected domain \( \Omega \) such that \( g(z) \neq 0 \) for \( z \in \Omega \). Let \( \gamma^* \) be a positively oriented circular path in \( \Omega \) which does not go through any of the poles of \( g \). Prove that
\[
\frac{1}{2\pi i} \int_\gamma \frac{g'(\xi)}{g(\xi)} \, d\xi = \sum_{\text{poles } p \text{ inside } \gamma} N(p),
\]
where \( N(p) \) is the order of a pole \( p \).

9. Suppose that \( f \in H(\Omega) \), a domain \( \Omega \) contains a closed unit disc, \(|f(z)| > 2\) for \(|z| = 1\), and \( f(0) = \sqrt{2} \). Must \( f \) have a zero in the unit disc?