HOMEWORK 3. DUE FRIDAY FEBRUARY 1

HAND IN:
6.3.7, 6.3.8, 6.3.9, 6.3.12, 6.4.2, 6.4.3. 6.4.4.

AND:

1. If A is a ring, an *anti-automorphism of* A is a bijection $\varphi\colon A \to A$ such that, for all $x, y \in A$, $\varphi(x+y) = \varphi(x) + \varphi(y)$ and $\varphi(xy) = \varphi(y)\varphi(x)$.

(i) Let $\mathbb{H}$ be the division ring of quaternions. For each $x = a{\cdot}1+b{\cdot}i+c{\cdot}j+d{\cdot}k \in \mathbb{H}$ write $\bar{x} = a{\cdot}1-b{\cdot}i-c{\cdot}j-d{\cdot}k$. Prove that the function $x \mapsto \bar{x}$ is an anti-automorphism of $\mathbb{H}$.

(ii) Construct an anti-automorphism φ of $\mathbb{R}Q$ (Hint: make φ compatible with the "bar" anti-automorphism of $\mathbb{H}$)

(iii) Construct an anti-automorphism of the ring $Mat_{n\times n}(\mathbb{R})$ of all $n\times n$ matrices with real coefficients. And prove it.

2. Prove: $\mathbb{Z}[\sqrt{2}] \cong \left\{ \begin{pmatrix} a & 2b \\ b & a \end{pmatrix} \;\middle|\; a, b \in \mathbb{Z} \right\}$ (the latter is a subring of the ring of 2×2 matrices over $\mathbb{Z}$).

DO NOT HAND IN:

6.3.1, 6.3.2, 6.3.3, 6.3.6, 6.3.10, 6.4.1