HOMEWORK 8. DUE FRIDAY NOVEMBER 30

HAND IN:
#1.7.3, 1.7.11, 1.7.12, 1.7.13, 3.1.3, 3.3.3, 3.3.4, 3.3.5, 3.3.9

AND

1. Prove that D_4 and the quaternion group Q are the only (up to isomorphism) NON-ABELIAN groups of order 8.

DO NOT HAND IN:
#1.7.1, 1.7.2, 1.7.4, 1.7.5, 3.1.2, 3.2.2, 3.2.3, 3.2.4, 3.3.1, 3.3.2, 3.3.10.

AND:

2. Denote by $\binom{n}{k}$ the binomial coefficient $\dfrac{n!}{k!(n-k)!}$ for $0 \le k \le n$. Let R be any commutative ring with identity element 1_R.

(i) Use INDUCTION to prove the Binomial Theorem in R: For all integers $n \ge 1$ and all $a, b \in R$,

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n-1}ab^{n-1} + b^n.$$

(ii) If $2 \cdot 1_R = 0$, show that $(a+b)^2 = a^2 + b^2$.

(iii) Generalize (ii) to the case in which $p \cdot 1_R = 0$ for a *prime* p.