# HOMEWORK 6. DUE FRIDAY NOVEMBER 9

HAND IN:

#2.5.4, 2.5.6, 2.5.10, 2.5.13, 2.5.15, 2.6.2, 2.6.5.

AND:

1. Find a subgroup $H$ of $S_3$ such that the correspondence $aH \mapsto Ha$, $a \in G$ does NOT define a bijection between the set of left cosets and the set of right cosets of $H$ in $G$. BETTER: find a subgroup $H$ of $S_3$ such that $aH \mapsto Ha$ does not even define a FUNCTION from the set of left cosets to the set of right cosets.

2. Let $Q$ be the subgroup of $GL_2(\mathbb{C})$ generated by the matrices $A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (The group $Q$ is called the *quaternion group*). Prove:
(i) $|Q| = 8$.
(ii) Every element of $Q \backslash \{\pm I_2\}$ has order 4.
(iii) $Q$ has EXACTLY 6 subgroups.
(iv) Every subgroup of $Q$ is normal, but $Q$ is not abelian.

3. If $G$ is an ABELIAN group of order 8 prove that $G$ is either cyclic or isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_4$ or isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$. You can use the following outline:
(i) Every element of $G$ has order 1, 2, 4, or 8; WHY?
(ii) Suppose there is an element $a$ of order 8. Then what can you say about $G$?
(iii) Now suppose that all elements of $G$ are of order 1 or 2. Show that $G \cong \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$.
(iv) Now suppose that $g^4 = e$ for all $g \in G$ and at least one element $a$ of $G$ is of order 4. Show that $G \cong \mathbb{Z}_2 \times \mathbb{Z}_4$.

DO NOT HAND IN #2.5.11, 2.5.14, 2.5.16, 2.6.1, 2.6.4.

AND

3. Prove that every finite group of order $> 2$ has an automorphism other than the identity automorphism.