HOMEWORK 4. DUE FRIDAY OCTOBER 26

HAND IN:

#2.4.5, 2.4.6, 2.4.8, 2.4.9, 2.4.12, 2.4.14.

AND

1. (i) For each $a \in \mathbb{Z}$ show that $\varphi_a\colon [x] \mapsto [ax]$ defines a homomorphism $\mathbb{Z}_n \to \mathbb{Z}_n$. (Is it well-defined?)

(ii) Prove: Ker $\varphi_a = m\mathbb{Z}_n$ where $m = n/\gcd(a, n)$.

2. TRUE, FALSE, and WHY?

(i) For elements a, b in a finite group, $|a^2| = |b^2| \Rightarrow |a| = |b|$.

(ii) There is a group containing elements a, b such that $|a| = 2, |b| = 7|, |ab| = 2$.

(iii) The subgroup of $\mathbb{Z}_n$ generated by $[k]$ is also generated by $[\gcd(k, n)]$.

DO NOT HAND IN:

#2.4.1-2.4.4, 2.4.15

AND

3. Use a dihedral group to give an example of groups $A \subset B \subset G$ such that A is normal in B and B is normal in G but A is not normal in G. Prove that your example behaves as stated.