HOMEWORK 3. DUE FRIDAY OCTOBER 19

HAND IN: #2.2.18, 2.2.24, 2.2.25, 2.2.27, 2.3.6, 4.4.2, 4.4.5.
AND:

1. (Heisenberg group) Prove that the set of matrices of the form $\begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}$ with $a, b, c \in \mathbb{R}$ is a group under multiplication. What if we only use $a, b, c \in \mathbb{Q}$? What if we only use $a, b, c \in \mathbb{Z}$?

2. If elements a and b of a group commute (i.e., satisfy $ab = ba$), prove the following (induction, please).
(i) $a^m b^n = b^n a^m \ \forall m, n \in \mathbb{Z}$.
(ii) $(ab)^n = a^n b^n \ \forall n \in \mathbb{Z}$.

DO NOT HAND IN:
#2.3.1-2.3.4, 4.3.1-4.3.3, 4.4.1, 4.4.3.
AND:
3. Prove: Every group G of order 6 is either cyclic or is isomorphic to S_3. You might use the following outline for your proof:
(i) Prove that G cannot have elements of order ≥ 4.
(ii) Prove that G has at least one element of order 2.
(iii) ASSUME there is just one element $a \in G$ of order 2. Prove: For any $b \in G \backslash \langle a \rangle$ either b or ab has order 6. So G is cyclic in this situation.
(iv) ASSUME that there are two elements a, b of order 2. Prove: ab does not have order 2. (Otherwise $\{1, a, b, ab\}$ would be a subgroup?). Then prove: $|ab| = 3$. Finally, work out the multiplication table for G and show that $(1, 2) \mapsto a$, $(2, 3) \mapsto b$, defines an isomorphism.