HOMEWORK 1. DUE FRIDAY OCTOBER 5

HAND IN:

EXCERCISES #1.3.2, 1.4.2, 1.10.4, 2.1.1, 2.1.7 (induction please), 2.1.12, 2.1.14

AND:

1. (i) Use induction and trigonometric identities to prove **DeMoivre's Theorem**: For any $\theta \in \mathbb{R}$ and any integer $n \geq 0$, $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$. (Please do NOT use the **Euler Formula**: $e^{i\theta} = \cos\theta + i\sin\theta$.)

(ii) Prove using induction: For any integer $n \geq 0$ and any $\theta \in \mathbb{R}$,

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}^n = \begin{pmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{pmatrix} .$$

What does this have to do with the rotation through θ about the origin?

2. Let G be a group and $a \in G$.

(i) Prove: for any integers $m, n \geq 0$, $a^m a^n = a^{m+n}$. (Induction please!)

(ii) Prove: for any integer $n \geq 0$, $(a^{-1})^n = a^{-n}$.

(iii) Prove: for any integers m, n, $a^m a^n = a^{m+n}$.

DO NOT HAND IN:

EXCERCISES #1.3.1, 1.3.3, 1.4.1, 1.10.2, 1.10.3. 2.1.2-2.1.5, 2.1.11 (after you've done 2.1.12)