

## MATH 241 FINAL EXAM STUDY GUIDE

### How to prepare for this exam:

1. Look over all of your old homework. Make sure that what you wrote down still makes sense.
2. Redo all of your quizzes. Even if you did them perfectly, redoing them will remind you about the different types of problems.
3. Read over your notes. There are many subtleties that are important to understand. Once you understand these then you understand the material much better.
4. Look over the Review section at the end of chapters 1, 9, 10, 11, 12.

Format for the final: **Multiple choice** (please bring pencil #2).

You will not be able to use your notes, or book on the exam. You can use one note card  $3 \times 5$  inches with any formulas you wish.

Review questions:

- Page 93: 3, 5, 7, 8, 9, 14, 15, 21, 23;
- Page 667: 1, 4, 9-11, 13, 15, 19, 21, 25-28, 29, 31, 33-35;
- Page 784: 1, 2, 5, 7, 9, 10, 11, 17, 21, 22, 23, 27, 29, 31, 33, 35, 37, 43, 45;
- Page 841: 1, 3, 5, 6, 7, 9, 11, 13, 15, 17, 19, 22, 23, 24;
- Page 911: 1, 3, 5, 7, 9-13, 15, 17, 23-28.

### Vocabulary:

- function
- piecewise function
- $X$  and  $Y$  intercept
- linear function
- slope
- difference quotient
- point-slope form
- quadratic function
- vertex
- concave up
- concave down
- cost, revenue, profit, supply, demand
- exponential function
- logarithmic function
- natural log
- continuously compounded interest
- half-life
- doubling time
- average rate of change
- difference quotient
- secant line through  $P$  and  $Q$
- instantaneous rate of change
- Derivative of  $f(x)$  at  $x = a$
- Limit of a function of  $f(x)$  at  $x = a$
- L'Hospital rule

- Tangent line to  $f(x)$  at  $x = a$
- Product Rule
- Quotient Rule
- Chain Rule
- Implicit differentiation
- Marginal cost
- Marginal revenue
- Marginal profit
- Marginal revenue
- Marginal product
- Relative and absolute minima and maxima
- Stationary points, singular points, endpoints
- Second derivative, acceleration
- Concavity, points of inflection

## 1. CHAPTER 1

Sections 1.1, 1.2, 1.3, 1.4. The most important sections are 1.3 and 1.4. Here are some things you should know:

- How to evaluate and graph piecewise functions.
- How to find the  $X$  and  $Y$  intercepts of a function.
- How to write the equation of a line in point-slope form.
- How to write the cost, revenue, profit, supply and demand functions and use these functions to answer application questions.
- Interpret the meaning of the slope for linear models.

## 2. CHAPTER 9

Sections 9.1, 9.2, 9.3: quadratic, exponential, and logarithmic models. You should know the basic form of all three models.

- Quadratic Models; basic form:  $f(x) = ax^2 + bx + c$ , where  $a \neq 0$ .

Given a quadratic function, be able to find the vertex.

Be able to answer minimum/maximum application questions especially concerning revenue and profit.

Be able to graph a quadratic function and use a graph to answer questions about a quadratic function.

- Exponential Models; basic form:  $f(x) = Ab^x$ ,  $b > 0$ ,  $b \neq 1$

Logarithmic Functions; You basically just need to know how to use logarithms to solve exponential equations. Its very important to understand the relationship between  $\log_b(x)$  and  $b^x$ .

Know the power laws!!

Be familiar with the investment models where interest is compounded continuously, annually, monthly, daily etc.

Know the difference between exponential decay and growth models.

Given two points on an exponential function you should be able to find the function.

Solve exponential equations using logarithmic functions (9.3).

Find doubling time or half-life using growth or decay constants (9.3).

Find the growth or decay constants using doubling time or half-life (9.3).

## 3. CHAPTER 10

Sections 10.1, 10.4, 10.5, 10.6, 10.7, 10.8. Limits, rate of change, and derivative. Here are some things you should know:

- Know how to compute limits using the L'Hospital rule.
- Be able to find the average rate of change of a function over an interval of the type  $[a, b]$  or  $[a, a + h]$ . You should be able to do this for functions given by equations, tables, or graphs.
- Know how to estimate the instantaneous rate of change of  $f(x)$  at  $x = a$ .
- Understand how average rate of change is related to the slope of secant lines of graph.
- Know to the definition of the derivative of a function using as the limit of difference quotient.
- You need to be able to find the derivative of a function using the limit of difference quotient.
- Understand how derivative is related to the slope of tangent lines of graph.
- Know how to use the derivative to find the equation of the tangent line to  $f$  at the point  $x = a$ .
- Know how to compute derivatives using the power, constant multiple, and the sum and difference rules.
- Understand the applications in section 10.8, in particular, the marginal product example given in the text.

## 4. CHAPTER 11

Sections 11.1, 11.2, 11.3, 11.4. Here are some things you should know:

- You need to know the product, quotient and chain rules and, obviously, how to use them.
- You need to know how to use the above rules in conjunction with one another. i.e. be able to compute the derivative of a quotient of products, the product of quotients, a product of compositions etc. Basically you to be able to figure out which rule to use first etc.
- Know the derivative of  $\log_b(x)$  and  $b^x$ . If you know these derivatives then you can let  $b = e$  to remember the derivatives of  $\ln(x)$  and  $e^x$ . You should also know the derivative of  $\log_b(|x|)$ .
- Be able to use the above derivatives to answer questions about rates of change of exponential and logarithmic questions.
- Be able to use the above rules for implicit differentiation.

## 5. CHAPTER 12

Sections 12.1, 12.2, 12.3. Here are some things you should know:

- You need to know how to locate relative and absolute minima and maxima on the graph.
- Be able to classify minima and maxima using stationary, singular point, and endpoints.
- Understand how the stationary and singular points are related to the derivative.
- Understand the steps in solving optimization problems, e.g., maximizing revenue, profit.
- Be able to find all extrema (i.e., minima and maxima) of a function on a closed interval.
- You need to understand the second derivative as the acceleration or, which is the same, as the rate of change of the derivative.
- Be able to use the second derivative to answer questions about concavity and inflection points.